Calculus Application Problem - Around the Corner

1. The Problem.
A steel pipe is being carried north up a north/south hallway that is 8 feet wide. At the end of the hall there is a right-angled turn into a wider hallway 12 feet wide. What is the length of the largest pipe that can be carried around the corner? Assume the pipe must remain horizontal and that it can not be tilted. We will not be concerned with the diameter of the pipe in computing a solution to the problem.

2. A Trial-and-Error Solution.
Using a piece of graph paper or engineering paper, draw a vertical hallway 8 units wide that makes a right-angled turn into a horizontal hallway 12 units wide.

First, without doing any measuring or calculations, what is your intuitive "guess" with respect to the maximum length of the pipe that will make it around the corner?

Approximately ____________ units.

With a piece of spaghetti representing the steel pipe, break small pieces off of the end until the spaghetti will be able to just fit around the corner. Try to find the length of the spaghetti to the nearest 0.5 unit. As you do this, try to determine some of the necessary properties of the pipe that will make it the "longest pipe to fit around the corner". (Hint: To help determine some of the properties, after you answer the problem, cut another piece off of the spaghetti and see how this pipe is different from the pipe that answers the problem.)

Record the length here. ____________ units. (Pipe is measured by units on the graph paper.)

What must be true about the pipe in relation to the two walls and the corner in order for us to find the length of the longest pipe that fits around the corner? ______________________________

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3. The Analytical Solution.
This problem can actually be solved analytically two different ways. One procedure involves trigonometry, while the other does not. We are going to solve this problem both ways, and we'll see if our results match and correspond to the "trial-and error" solution.

a) Solution #1 (Without trigonometry)
Draw a straight line on your figure representing the pipe. It should connect the two outside walls and touch the inside corner. Now, on your figure, draw lines to extend the inside walls of the hallways to the opposite sides. Your figure should contain two **similar triangles** and a **rectangle**. Label the lengths of the segments on the outside walls that we know (8 and 12) and also the parts that we don't know (with an X for the horizontal segment and a Y for the vertical segment). If the steel pipe measures a total of **L** feet, the figure shows it divided into two parts. Label theses two parts of the pipe **L1** and **L2**.

Using the Pythagorean Theorem write an equation containing **L1** and an equation containing **L2**. Solve for **L1** and **L2** and write your expressions below.

L1 = ____________________ **L2** = ____________________

We need both of the above expressions in terms of the same variable **X**. Using the geometry of **similar triangles**, write a proportion which shows the relationship between **X** and **Y**.

Solve the equation for **Y** and complete the expression below, which shows the length of the pipe as a function of **X**.

L(X) = L1 + L2 = ______________________________

On your calculator, graph **L** and use your graph to find the **minimum value** of the length of the pipe. (Remember, although it is a minimum point on your graph, it is the maximum length of pipe that will fit around the corner!)
Sketch your graph below and label your axes to show the WINDOW that you used.

The pipe should be
__________ units.

How does your analytical answer compare to you **"trial-and-error"** answer using the piece of spaghetti?
____

Let's verify this answer using calculus. We know that local minimums (and maximums) of a function occur when ____.

On another sheet of paper, show all of the analytical work to solve the problem. Be neat! This step is a major part of this problem!

b) Solution #2 (With trigonometry)

Label the angle in the bottom left corner of the your figure θ. Another angle in your figure can also be labeled θ. Find it and label it.

Using your right triangle trigonometry relationships, write a trigonometric equation for the relationship between **L1** and θ, and an equation between **L2** and θ. (Do not use X and Y in your equations.) Solve the equations for **L1** and **L2** and write your equations below.

L1 = ____________________ **L2 =** ____________________

Again, the total length of the pipe is the sum of **L1** and **L2**. This time, however, the length **L** is a function of the angle θ. Complete the equation below.

L(θ) = L1 + L2 = ____

Again, we will solve this both **graphically** and **analytically**. On your calculator, graph **L**, and use your graph to find the minimum value of the length of the pipe. Of course, substitute **X** for the θ in your equation. Since we are going to apply our calculus on this result as well, leave your calculator in RADIAN mode but adjust your window to see a reasonable graph.

(What values of **X** make sense to the problem? ____________________

Sketch your graph below and label your axes to show the WINDOW that you used.

The pipe should be

________ units.

How does this answer compare to your answer from **Solution #1**? ____________________

Again, let's verify this answer using the calculus and algebra necessary to solve the problem.
Show all of your work again on the other paper.